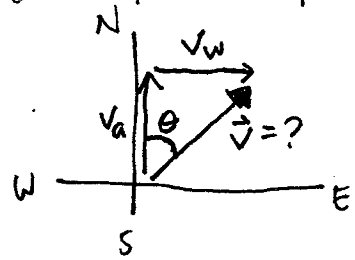
Problem 1 An airplane flies at speed $v_a = 150$ mi/hr relative to the air. A wind is blowing at speed $v_w = 120$ mi/hr toward the east, relative to the ground. (a) If the plane flies due north according to its onboard compass, find the speed and direction (relative to north) of the plane as measured by observers on the ground. (b) If the plane flies due north according to ground observers, how fast does the plane move relative to the ground? In this case, at what angle to north must the pilot fly, according to the onboard compass?

→ observers on ground = frame S
observer moving with the air (wind) = frame S′
Air (S′) moves with respect to ground (S) at $\vec{V} = V_w = 120$ mph, E.
Plane moves at $V_a = 150$ mph relative to air, so $\vec{v}' = 150$ mph, some direction.

(a) Here we are told $\vec{v}'$ is to the north.
We are asked for the plane's velocity as measured by observers on the ground → this is asking us for $\vec{v}$, the plane's velocity as measured by observers in frame S.
Galilean velocity transform $\vec{v}' = \vec{v} - \vec{V}$, so $\vec{v} = \vec{v}' + \vec{V}$.

Use vector addition:

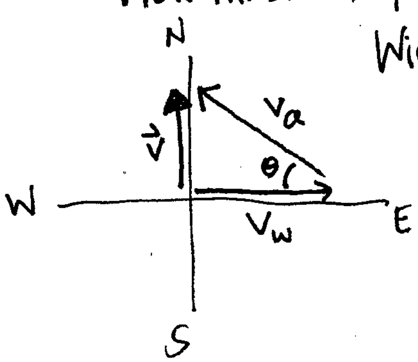


$$v = \sqrt{V_a^2 + V_w^2} = \sqrt{(150\,\text{mph})^2 + (120\,\text{mph})^2} = 190\,\text{mph}$$

$$\theta = \tan^{-1}\left(\frac{V_w}{V_a}\right) = \tan^{-1}\left(\frac{120}{150}\right) = 39° \text{ East of North}$$

$\boxed{\vec{v} = 190\,\text{mph}, \text{ ~~East~~ at } 39° \text{ E of N}}$

(b) Different situation: now we're told $\vec{v}$ = some speed, north.
How must the pilot fly relative to air to move north relative to ground?



Wind blows plane east, so $\vec{v}'$ must be northwest somehow to make $\vec{v}$ purely north (see diagram).

$$|\vec{v}| = v = \sqrt{V_a^2 - V_w^2} = 90\,\text{mph}$$

$$\theta = \cos^{-1}\left(\frac{V_w}{V_a}\right) = 37°$$

⇒ 37° from W or 53° from N

Plane moves north rel. to ground at 90 mph.

Pilot must fly at 53° W of N relative to the air.